Midterm, Math100, November 6, 2014
(R. Boltje)

Show all your work. No calculator, cell phones, electronic devices, textbooks, or notes allowed.

Maximum score: 30 points

Time: 1 hour 45 minutes.

1. (6 points) Write the set $A = \{X \in \mathcal{P}(\{1, 2, 3\}) \mid 2 \in X\}$ explicitly as $A = \{., \ldots, .\}$ by listing its elements.

2. (6 points) Let $\mathbb{P}$ denote the set of prime numbers. Negate the statement

$$\forall n \in \mathbb{N}, \ \exists p, q \in \mathbb{P}, \ 2(n+1) = p + q\,.$$

3. (6 points) Count the number of 4-digit integers (from 1000 to 9999) which are even or don't contain the number 6 as digit. Justify your answer.

4. (6 points) Show that for every $n \in \mathbb{N}$ with $n \geq 2$ one has $n^2 = 2\binom{n}{2} + \binom{n}{1}$.

5. (6 points) Let $a, b \in \mathbb{Z}$. Show that if $4 \mid a^2 + b^2$ then a and b are even.