Solutions to Practice Midterm, Version 1 (showing all the work)

1. (a) $\int_{-1}^{1} x^3 dx = \frac{x^4}{4}|_{-1}^{1} = \frac{1}{4} - \frac{1}{4} = 0.$

(b) $\int_{-\pi}^{\pi} \cos(\theta) d\theta = \sin(\theta)|_{-\pi}^{\pi} = \sin(\pi) - \sin(-\pi) = 0 - 0 = 0.$

(c) Use the substitution $u(t) = \sqrt{t}$. Then $u'(t) = 1/2\sqrt{t}$ and we obtain

$$\int_1^2 \frac{e^{\sqrt{t}}}{\sqrt{t}} dt = 2\int_1^2 e^{u(t)} u'(t) dt = 2\int_{\sqrt{1}}^{\sqrt{2}} e^u du = 2e^u|_1^{\sqrt{2}} = 2e^{\sqrt{2}} - 2e\,.$$

2. Sketch of the curves: ....

Intersection of curves $y = e^x$ and $y = 2$: $e^x = 2 \iff x = \ln 2$. Thus, only one intersection at $(\ln 2, 2)$.

Intersection of curves $y = e^x$ and $y = 1 - x$: $e^x = 1 - x$ holds for $x = 0$ and for no other x (for $x > 0$ we have $e^x > 1$ and $1 - x < 1$, and for $x < 0$ we have $e^x < 1$ and $1 - x > 1$). Thus, only one intersection at $(0, 1)$.

Intersection of curves $y = 2$ and $y = 1 - x$: $2 = 1 - x$ has only one solution, namely $x = -1$. Only one intersection at $(-1, 2)$.

The area between of the region is

$$\int_{-1}^{0} (2 - (1 - x))dx + \int_0^{\ln 2} (2 - e^x)dx = \int_{-1}^{0} (1 + x)dx + \int_0^{\ln 2} (2 - e^x)dx$$

$$= (x + \frac{x^2}{2})|_{-1}^{0} + (2x - e^x)|_0^{\ln 2} = -(-1 + \frac{1}{2}) + (2\ln 2 - 2) - (0 - 1) = -\frac{1}{2} + 2\ln 2\,.$$

3. By the fundamental theorem of calculus one has $A'(x) = e^{x^2}$.

4. Use substitution $u(x) = x^2$. Then $u'(x) = 2x$ and

$$\int x \tan(x^2) dx = \frac{1}{2}\int u'(x)\tan(u(x))dx = \frac{1}{2}\int \tan(u) du\,.$$

If you know the antiderivative of $\tan(u)$ you can use it.

If not continue as follows: Write $\tan(u) = \sin(u)/\cos(u)$ and use the substitution $v(u) = \cos(u)$ with $v'(u) = -\sin(u)$. then

$$\int \frac{\sin(u)}{\cos(u)} d(u) = -\int \frac{1}{v} dv = -\ln|v| + C \ = \ln|\cos(u)| + C\,.$$

Altogether:

$$\int x \tan(x^2) dx = -\frac{1}{2}\ln|v| + C = -\frac{1}{2}\ln|\cos(x^2)| + C\,.$$

Solutions to Practice Midterm, Version 2 (result only)

1. $\pi/2$.

2. $4\pi R^3/3$ (with disc or shell method).

3. $\int \frac{t}{\sqrt{4-t^2}} dt = -\sqrt{4-t^2} + C$ (with substitution $u(t) = 4 - t^2$).

4. $F(x) = x\sin(x) + \cos(x)$ (integrating by parts).

Solutions to Practice Midterm, Version 3 (result only)

1. $2/3$.

2. $\pi/4$.

3. 8π.

4. $(x^2 - 2x - 2)e^x + C$ (integration by parts twice).